Übungen

7. e) $4v^2 + 4v + 1 = (2v+1)^2$

$a^2 + \dots + b^2$

$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$(a+b)(a-b) = a^2 - b^2$

f) $9s^2 - 18s + 9 = \underline{(3s-3)^2}$

$a^2 - \dots + b^2 =$

$\hookrightarrow 9s^2 - 18s + 9$

Bruchterme

1. a) $\frac{1}{x-1}$ $D = \mathbb{R} \setminus \{1\}$

c) $\frac{x+1}{x}$ $D = \mathbb{R} \setminus \{0\}$

e) $\frac{1}{x+2} + \frac{1}{x}$ $D = \mathbb{R} \setminus \{-2, 0\}$

GN: $(x+2) \cdot x$

$\frac{1}{3} + \frac{1}{7}$

$\frac{1 \cdot x}{(x+2) \cdot x} + \frac{1 \cdot (x+2)}{(x+2) \cdot x}$

$\frac{7}{21} + \frac{3}{21}$

$= \frac{x + (x+2)}{(x+2) \cdot x}$

$= \frac{7+3}{21}$

$= \underline{\frac{2x+2}{(x+2) \cdot x}}$

Bsp. $\frac{x^2 - 3x}{x \cdot (x-3) \cdot 2} = \frac{\cancel{x}\cancel{(x-3)}}{\cancel{x}\cancel{(x-3)} \cdot 2} = \underbrace{\frac{x(x-3)}{x(x-3)}}_{1} \cdot \frac{1}{2}$

k) $\frac{1}{x+1} + \frac{1}{x^2+1}$ $\qquad D = \mathbb{R} \setminus \{-1\}$

u) $\frac{2a - \frac{1}{a+1}}{a}$ $\qquad D = \mathbb{R} \setminus \{0, -1\}$

$$= \frac{\frac{2a}{1} - \frac{1}{a+1}}{a} = \frac{\frac{2a(a+1)}{a+1} - \frac{1}{a+1}}{a}$$

$$= \frac{\frac{2a(a+1)-1}{a+1}}{\frac{a}{1}} = \frac{2a(a+1)-1}{a+1} : \frac{a}{1}$$

$$= \frac{2a(a+1)-1}{a+1} \cdot \frac{1}{a} = \frac{(2a(a+1)-1) \cdot 1}{(a+1) \cdot a}$$

$$= \frac{2a(a+1)-1}{(a+1) \cdot a}$$

3. j) $\frac{2m}{3k^2} + \frac{3}{k} = \frac{2m}{3k^2} + \frac{3 \cdot 3k}{k \cdot 3k} = \frac{2m}{3k^2} + \frac{9k}{3k^2}$

GN: $3k^2 = 3 \cdot k \cdot k$ $\qquad = \frac{2m + 9k}{3k^2}$

$k = k$

kgV: $3 \cdot k^2$

Bsp. kgV$(60, 80) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

60	2
30	2
15	3
5	5
1	

80	2
40	2
20	2
10	2
5	5
1	

o) $\frac{1}{x} - \frac{1}{y} + \frac{1}{2}$ GN: $2xy$

$$= \frac{2y}{2xy} - \frac{2x}{2xy} + \frac{xy}{2xy} = \frac{2y-2x+xy}{2xy}$$

4. i) $\frac{3}{2x+2y} - \frac{1}{x+y}$ GN: $2x+2y = 2(x+y)$

$x+y$

GN: $2\cdot(x+y)$

$$= \frac{3}{2(x+y)} - \frac{1}{x+y}$$

$$= \frac{3}{2(x+y)} - \frac{2}{2(x+y)} = \frac{3-2}{2(x+y)} = \frac{1}{2(x+y)}$$

$$\frac{3}{2x+2y} - \frac{1}{x+y} = \frac{3(x+y)}{(2x+2y)(x+y)} - \frac{1(2x+2y)}{(2x+2y)(x+y)}$$

$$= \frac{3(x+y)-(2x+2y)}{(2x+2y)(x+y)} = \frac{3x+3y-2x-2y}{(2x+2y)(x+y)}$$

$$= \frac{\cancel{x+y}}{(2x+2y)\cancel{(x+y)}} = \frac{1}{2x+2y}$$

$a^2 - b^2 = (a+b)(a-b)$

k) $\frac{m}{m^2-1} - \frac{1}{m-1}$ GN: $m^2 - 1^2 = (m+1)(m-1)$

$m-1$

$$\frac{m}{(m+1)(m-1)} - \frac{1}{m-1}$$

$$= \frac{m}{(m+1)(m-1)} - \frac{m+1}{(m-1)(m+1)} = \frac{m-(m+1)}{(m-1)(m+1)}$$

$$= \frac{m-m-1}{(m-1)(m+1)} = \frac{-1}{(m-1)(m+1)} = -\frac{1}{(m-1)(m+1)}$$

6. c) $\frac{4u}{3v^2} \cdot 6v \cdot \frac{1}{u} = \frac{4u}{3v^2} \cdot \frac{6v}{u} = \frac{\cancel{24}^{8}\cancel{u}\cancel{v}}{\cancel{3}\cancel{u}v^{\cancel{2}}} = \frac{8}{v}$

$$\frac{8}{v^1} = \underline{8 \cdot v^{-1}}$$

! $\boxed{\frac{1}{a^n} = a^{-n}}$

f) $\left(2 - \frac{1}{k}\right) \cdot \left(2 + \frac{1}{k}\right) = \left(\frac{2k}{k} - \frac{1}{k}\right) \cdot \left(\frac{2k}{k} + \frac{1}{k}\right)$

$$= \left(\frac{2k-1}{k}\right)\left(\frac{2k+1}{k}\right) = \frac{2k-1}{k} \cdot \frac{2k+1}{k}$$

$$= \frac{\overset{(a-b)\ (a+b)}{(2k-1)(2k+1)}}{k \cdot k} = \frac{\overset{a^2 - b^2}{4k^2 - 1}}{k^2} = \frac{4\cancel{k^2}}{\cancel{k^2}} - \frac{1}{k^2}$$

$$\underline{= 4 - \frac{1}{k^2}}$$

kürzer: $\overset{(a-b)}{\left(2 - \frac{1}{k}\right)}\overset{(a+b)}{\left(2 + \frac{1}{k}\right)} = 4 - \left(\frac{1}{k}\right)^2$

$$= 4 - \frac{1^2}{k^2}$$

$$= \underline{\underline{4 - \frac{1}{k^2}}}$$

g) $m \cdot n \cdot \left(\frac{1}{m} - \frac{1}{n}\right) = m \cdot n \cdot \left(\frac{n}{m \cdot n} - \frac{m}{m \cdot n}\right)$

$$= m \cdot n \cdot \left(\frac{n-m}{m \cdot n}\right)$$

$$= \cancel{m} \cdot \cancel{n} \cdot \frac{n-m}{\cancel{m} \cdot \cancel{n}} = \underline{n - m}$$

$$\left(= \frac{\cancel{m} \cdot \cancel{n} \cdot (n-m)}{\cancel{m} \cdot \cancel{n}} \right)$$

v) $\frac{p-3q}{q^2} : \frac{2p-6q}{q} = \frac{p-3q}{q^2} \cdot \frac{q}{2p-6q} = \frac{(p-3q)\cdot \cancel{q}}{q^{\cancel{2}}\cdot(2p-6q)}$

$= \frac{p-3q}{q\cdot(2p-6q)} = \frac{\cancel{p-3q}}{q\cdot 2\cdot \cancel{(p-3q)}} = \frac{1}{2q}$

7. e) $\frac{\frac{4r^2-1}{s}}{\frac{2r-1}{s^2}} = \frac{4r^2-1}{s} : \frac{2r-1}{s^2} = \frac{4r^2-1}{s} \cdot \frac{s^2}{2r-1}$

$a^2 - b^2 = (a-b)(a+b)$

$= \frac{(4r^2-1)\cdot s^{\cancel{2}}}{\cancel{s}\cdot(2r-1)} = \frac{(4r^2-1)\cdot s}{2r-1}$

$= \frac{\cancel{(2r-1)}(2r+1)\cdot s}{\cancel{2r-1}} = \underline{(2r+1)\cdot s} = 2rs+s$

Bsp.: $\frac{\frac{\frac{x}{4}}{2x}}{5} = \frac{\frac{x}{4}}{2x} : \frac{5}{1} = \frac{\frac{x}{4}}{\frac{2x}{1}} \cdot \frac{1}{5}$

$= \left(\frac{x}{4} : \frac{2x}{1}\right) \cdot \frac{1}{5}$

$= \frac{x}{4} \cdot \frac{1}{2x} \cdot \frac{1}{5}$

$= \frac{\cancel{x}}{40\cancel{x}} = \frac{1}{40}$

Gleichungen

$$3x^2 - 4x + 5 = 2x^2 - 5x - 1$$

$$\frac{1}{x-1} = \frac{1}{x} + x + 1$$

$$3x - 5 = 4(2x - 4)$$

$$3x - 5 = 8x - 16 \quad | +16$$

$$3x - 5 + 16 = 8x - 16 + 16$$

$$3x + 11 = 8x \quad | -3x$$

$$11 = 5x$$

$$5x = 11 \quad | :5$$

$$\underline{x = \frac{11}{5}} = 2{,}2$$

$G = \mathbb{R}$

$D = \mathbb{R}$

$L = \left\{ \frac{11}{5} \right\}$

Lösungsfälle

eindeutige Lösung	keine Lösung	allgem. gültige Lösung
$x + 3 = 5$	$x = x + 1$	$x = x$
$x = 2$	$0 = 1$	$0 = 0$
$L = \{2\}$	$L = \emptyset$	$L = \mathbb{R}$

Übungen

$G = \mathbb{N}$!

1. b) $14 - (x-9) = 10 + 5x - (7x-5)$

$14 - x + 9 = 10 + 5x - 7x + 5$

$23 - x = 15 - 2x \quad | -15$

$8 - x = -2x \quad | +x$

$8 = -x \quad | \cdot (-1)$

$-8 = x$

$\underline{x = -8}$ streng: $L = \emptyset$, weil $G = \mathbb{N}$

a) $2x - 18 + 8x + 1 = 20 + 5x - 2$

$10x - 17 = 5x + 18 \quad | +17 \quad | -5x$

$5x = 35 \quad | :5$

$\underline{x = 7}$ $\quad L = \{7\}$

l.S.: $2 \cdot 7 - 18 + 8 \cdot 7 + 1 = 14 - 18 + 56 + 1 = \underline{53}$

r.S.: $20 + 5 \cdot 7 - 2 = 20 + 35 - 2 = \underline{53}$ ✓

2. c) $\frac{2-x}{6} - \frac{x-3}{7} = 2 \quad | \cdot 6$

$6 \cdot \frac{2-x}{6} - 6 \cdot \frac{x-3}{7} = 12$

$2 - x - \frac{6x-18}{7} = 12 \quad | \cdot 7$

$2 \cdot 7 - 7 \cdot x - 7 \cdot \frac{6x-18}{7} = 12 \cdot 7$

$14 - 7x - (6x - 18) = 84$

$14 - 7x - 6x + 18 = 84$

$-13x + 32 = 84 \quad | -32$

$-13x = 52 \quad | :(-13)$

$\underline{x = -4}$

a) $\frac{1}{4} \cdot x = 9 - \frac{x}{5} \quad | \cdot 4$

$x = 36 - \frac{4x}{5} \quad | \cdot 5$

$5x = 180 - 4x \quad | +4x$

$9x = 180 \quad | : 9$

$\underline{x = 20}$

3. c) $\frac{x+3}{3} - \frac{x-1}{2} = \frac{9-x}{6} \quad | \cdot 3$

$x+3 - \frac{3(x-1)}{2} = \frac{3 \cdot (9-x)}{6} \quad | \cdot 2$

$2x+6 - 3(x-1) = \frac{\cancel{2 \cdot 3} \cdot (9-x)}{\cancel{6}}$

$2x+6-3x+3 = 9-x$

$-x+9 = 9-x$

$9-x = 9-x \quad | +x$

$\underline{9 = 9}$

wahre Aussage

allgemein gültige Gleichung, d.h. $L = \mathbb{R}$

d) $\frac{2x+3}{3} + \frac{x-1}{5} = \frac{13}{15} \cdot x \quad | \cdot 3$

$2x+3 + \frac{3(x-1)}{5} = \frac{3 \cdot 13}{15} \cdot x \quad | \cdot 5$

$10x + 15 + 3(x-1) = \frac{\cancel{3} \cdot \cancel{5} \cdot 13}{\cancel{15}} \cdot x$

$10x + 15 + 3x - 3 = 13x$

$13x + 12 = 13x \quad | -13x$

$12 = 0 \quad ?$

falsche Aussage

$\rightarrow$ keine Lösung, $L = \emptyset$