Übungen Potenzen

9. e) $\left(\frac{x^2}{y}\right)^{-1} = \frac{(x^2)^{-1}}{y^{-1}} = \frac{x^{-2}}{y^{-1}} = \frac{\frac{1}{x^2}}{\frac{1}{y}} = \frac{y}{x^2}$ 5. Potenzgesetz: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

11. a) $\frac{(2x)^2}{5x^3} \cdot \frac{6x}{5^{-1}} = \frac{2^2 \cdot x^2 \cdot 6x}{5x^3 \cdot 5^{-1}} = \frac{24x^3}{x^3} = 24$

$5 \cdot 5^{-1} = 5^0 = 1$

$(a \cdot b)^n = a^n \cdot b^n$

$(a \pm b)^n \neq a^n \pm b^n$

$\boxed{a^{-n} = \frac{1}{a^n}}$ $\boxed{a^{\frac{1}{n}} = \sqrt[n]{a}}$

weil $a^{-n} = a^{0-n} = \frac{a^0}{a^n} = \frac{1}{a^n}$

$x^0 = 1, \quad x \neq 0$

$0^n = 0, \quad n \neq 0$

$0^0 = 1$

Bsp. $\frac{24x + x^2}{x^3} = \frac{x(24+x)}{x \cdot x^2}$

$= \frac{x}{x} \cdot \frac{24+x}{x^2}$

$= \frac{24+x}{x^2}$

Wurzeln

1. b) $\sqrt{7} \cdot \sqrt{7} - \sqrt{7^2}$

$= \sqrt{7 \cdot 7} - \sqrt{7^2}$

$= \sqrt{7^2} - \sqrt{7^2}$

$= 0$

Alternative: $\sqrt{7} \cdot \sqrt{7} - \sqrt{7^2}$

$= (\sqrt{7})^2 - \sqrt{7^2}$

$= 0$

Frage: $(\sqrt[n]{7})^2 = \sqrt[n]{7^2}$

$\left(7^{\frac{1}{n}}\right)^2 = 7^{\frac{2}{n}}$

$\sqrt[n]{7^2} = (7^2)^{\frac{1}{n}} = 7^{\frac{2}{n}}$

f) $(1+\sqrt{2})^2 = 1^2 + 2 \cdot 1 \cdot \sqrt{2} + (\sqrt{2})^2 = 1 + 2\sqrt{2} + 2 = 3 + \sqrt{2^2} \cdot \sqrt{2}$

$(a+b)^2 = a^2 + 2ab + b^2$

$= 3 + \sqrt{8}$

$2 \cdot \sqrt[3]{4} = \sqrt[3]{2^3} \cdot \sqrt[3]{4} = \sqrt[3]{2^3 \cdot 4} = \sqrt[3]{32}$

$\sqrt[3]{56} = \sqrt[3]{8 \cdot 7} = \sqrt[3]{8} \cdot \sqrt[3]{7} = 2 \cdot \sqrt[3]{7}$

2 g) $\sqrt[2]{\sqrt[2]{3^4}} = \sqrt{(3^4)^{\frac{1}{2}}} = \left((3^4)^{\frac{1}{2}}\right)^{\frac{1}{2}} = \underbrace{(3^4)^{\frac{1}{4}}}_{\sqrt[4]{3^4}} = 3^1 = 3$

$\sqrt[3]{\sqrt[4]{x}} = \sqrt[3]{x^{\frac{1}{4}}} = (x^{\frac{1}{4}})^{\frac{1}{3}} = x^{\frac{1}{12}} = \sqrt[12]{x}$

e) $\sqrt[6]{5^4} = (5^4)^{\frac{1}{6}} = 5^{\frac{\cancel{4}\,2}{\cancel{6}\,3}} = 5^{\frac{2}{3}} = (5^2)^{\frac{1}{3}} = \sqrt[3]{5^2}$

e) $\sqrt[3]{\frac{1}{t^2}} = \left(\frac{1}{t^2}\right)^{\frac{1}{3}} = \frac{1^{\frac{1}{3}}}{(t^2)^{\frac{1}{3}}} = \frac{1}{t^{\frac{2}{3}}} = t^{-\frac{2}{3}}$

7. u) $(a-b)\cdot\sqrt{\frac{a+b}{a-b}} = \sqrt{(a-b)^2}\cdot\sqrt{\frac{a+b}{a-b}}$

$= \sqrt{\frac{(a-b)^2}{1}\cdot\frac{a+b}{a-b}} = \sqrt{\frac{(a-b)^2(a+b)}{(a-b)}} = \sqrt{(a-b)(a+b)}$

$= \underline{\underline{\sqrt{a^2-b^2}}}$

$(a+b)^2 \neq a^2+b^2$

$\sqrt{a^2+b^2} \neq a+b$

$(a-b)^2 = a^2-2ab+b^2$

Bruchterme

4. e) $\frac{s-1}{s+1} - \frac{1}{s^2-1} = \frac{(s-1)(s-1)}{(s+1)(s-1)} - \frac{1}{(s+1)(s-1)} = \frac{(s-1)^2-1}{(s+1)(s-1)} = \frac{(s^2-2s+1)-1}{(s+1)(s-1)}$

$= \frac{s^2-2s}{(s+1)(s-1)} = \frac{s\cdot(s-2)}{(s+1)(s-1)}$

1. Nenner: $s+1$

2. Nenner: $s^2-1^2 = (s+1)(s-1)$

gem. Nenner $(s+1)(s-1)$

m) $\frac{1}{a-b} - \frac{1}{(a-b)^2} + \frac{1}{a^2-b^2}$

1. Nenner: $a-b$
2. Nenner: $(a-b)^2 = (a-b)(a-b)$
3. Nenner: $a^2-b^2 = (a-b)(a+b)$

$= \frac{(a-b)(a+b)}{gN} - \frac{(a+b)}{gN} + \frac{(a-b)}{gN}$

gN: $(a-b)^2(a+b)$

$= \frac{(a-b)(a+b)-(a+b)+(a-b)}{(a-b)^2(a+b)}$

$= \frac{a^2-b^2-a-b+a-b}{(a-b)^2(a+b)} = \frac{a^2-b^2-2b}{(a-b)^2(a+b)}$

o) $\frac{4x-2}{x^2-1} - \frac{3}{1+x} + \frac{1}{x-1} - \frac{2}{1-x}$

1. Nenner : $x^2-1 = (x-1)(x+1)$
2. N. : $1+x$
3. N $x-1$
4. N $1-x = -(x-1)$

$$\frac{4x-2}{x^2-1} - \frac{3}{x+1} + \frac{1}{x-1} - \frac{2}{-(x-1)}$$

$$= \frac{4x-2}{x^2-1} - \frac{3}{x+1} + \frac{1}{x-1} + \frac{2}{x-1}$$

gN: 1. $x^2-1 = (x+1)(x-1)$

$$= \frac{4x-2}{(x+1)(x-1)} - \frac{3(x-1)}{(x+1)(x-1)} + \frac{x+1}{(x+1)(x-1)} + \frac{2(x+1)}{(x+1)(x-1)}$$

$$= \frac{4x-2-(3x-3)+(x+1)+(2x+2)}{(x+1)(x-1)} = \frac{4x-2-3x+3+x+1+2x+2}{(x+1)(x-1)}$$

$$= \frac{4x+4}{(x+1)(x-1)} = \frac{4\cdot(x+1)}{(x+1)(x-1)} = \frac{4}{x-1}$$

Gleichungen

$$3x^2 - 5\sqrt{x} + 10 = 1 - 5\sqrt{x} + 3x^2 - x$$

Grundmenge = Zahlen im Kontext sinnvoll
Def. menge = — " — mathematisch sinnvoll
Lösungsmenge = Zahlen die die Gleichung wahr werden lassen

Bsp.: $\frac{6}{t} = 2$ t ... Zeit

$G = \mathbb{R}_0^+ = \{x \in \mathbb{R} : x \geq 0\}$

$D = \mathbb{R} \setminus \{0\}$

$L = \{3\}$

$x + 2 = 5 \quad | -2$ Äquivalenzumformung

$x + 2 - 2 = 5 - 2$

$x = 3$

Lösungsfälle:

1. eindeutige Lösung $x + 2 = 5 \rightarrow x = 3$
2. keine Lösung $x + 1 = x \rightarrow L = \emptyset = \{\}$
3. allgemeingültig (unendl. viele Lösungen) $x = x \rightarrow L = \mathbb{R}$

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1. c) $(x+1)\cdot(4x-3) = 2(x+1)\cdot(2x+3)$

$$4x^2 - 3x + 4x - 3 = 4x^2 + 6x + 4x + 6 \quad | -4x^2$$

$$-3x + 4x - 3 = 6x + 4x + 6$$

$$x - 3 = 10x + 6 \quad | -x$$

$$-3 = 9x + 6 \quad | -6$$

$$-9 = 9x \quad | :9$$

$$\underline{x = -1}$$

eig keine Lösung, weil $G = \mathbb{N}$!

2. c) $\frac{2-x}{6} - \frac{x-3}{7} = 2 \quad | \cdot 6$

$$\cancel{6} \cdot \frac{2-x}{\cancel{6}} - 6 \cdot \frac{x-3}{7} = 12$$

$$2 - x - \frac{6(x-3)}{7} = 12 \quad | \cdot 7$$

$$2 \cdot 7 - 7 \cdot x - \cancel{7} \cdot \frac{6 \cdot (x-3)}{\cancel{7}} = 84$$

$$14 - 7x - 6(x-3) = 84$$

$$14 - 7x - 6x + 18 = 84$$

$$-13x + 32 = 84 \quad | -32$$

$$-13x = 52 \quad | : (-13)$$

$$\underline{x = -4}$$

$$L = \{-4\}$$

3. c) $\frac{x+3}{3} - \frac{x-1}{2} = \frac{9-x}{6} \quad |\cdot 3$

$(x+3) - \frac{3(x-1)}{2} = \frac{3\cdot(9-x)}{6} \quad |\cdot 2$

$2(x+3) - 3(x-1) = \frac{\cancel{2}\cdot\cancel{3}\cdot(9-x)}{\cancel{6}}$

$2x+6-3x+3 = 9-x$

$9-x = 9-x \quad |+x$

$9 = 9 \quad |-9$

$0 = 0$

$L = \mathbb{R}$

d) $\frac{2x+3}{3} + \frac{x-1}{5} = \frac{13}{15}\cdot x \quad |\cdot 3$

$(2x+3) + \frac{3(x-1)}{5} = \frac{3\cdot 13}{15}\cdot x \quad |\cdot 5$

$5(2x+3) + 3(x-1) = \frac{\cancel{3}\cdot\cancel{5}\cdot 13}{\cancel{15}}\, x$

$10x+15+3x-3 = 13x$

$13x+12 = 13x \quad |-13x$

$12 = 0$

$\rightarrow L = \emptyset$

e) $\frac{x-1}{2} + \frac{x-2}{3} + \frac{x-3}{4} = x \quad |\cdot 2$

$x-1 + \frac{2(x-2)}{3} + \frac{\cancel{2}(x-3)}{\cancel{4}\,2} = 2x \quad |\cdot 3$

$3x-3+2x-4+\frac{3(x-3)}{2} = 6x \quad |\cdot 2$

$6x-6+4x-8+3x-9 = 12x$

$13x-23 = 12x \quad |-13x$

$-23 = -x \quad |\cdot(-1)$

$\underline{x = 23}$

f) $\frac{x-\frac{12-5x}{3}}{2}=\frac{1-x+\frac{x+4}{2}}{3} \quad |\cdot 2$

$x-\frac{12-5x}{3}=\frac{2(1-x+\frac{x+4}{2})}{3} \quad |\cdot 3$

$3(x-\frac{12-5x}{3})=2\cdot(1-x+\frac{x+4}{2})$

$3x-(12-5x)=2-2x+(x+4)$

$3x-12+5x=6-x$

$8x-12=6-x \quad |+x$

$9x-12=6 \quad |+12$

$9x=18 \quad |:9$

$\underline{x=2}$